

HW#4. 2.38. 6.13. 6.14. 6.20. 6.32. 6.41

2.38. Number of ways N_A molecule type A distribute on N sites.

$$8/8 \quad \Omega = \binom{N}{N_A} \quad 2$$

$$\Delta S_{\text{mixing}} = k \ln \Omega = k \ln \binom{N}{N_A} \quad 2$$

When $N \gg 1$ $N_A \gg 1$ $(N - N_A) \gg 1$

$$\begin{aligned} \Delta S_{\text{mixing}} &= k [\ln N! - \ln N_A! - \ln (N - N_A)!] \\ &= k [N \ln N - N_A \ln N_A - (N - N_A) \ln (N - N_A)] \end{aligned}$$

Replace N_A as xN $N - N_A$ as $(1-x)N$

$$\begin{aligned} \Delta S_{\text{mixing}} &= k [N \ln N - xN \ln N - xN \ln x - (1-x)N \ln(1-x) - (1-x)N \ln N] \\ &= k \cancel{N \ln N} \cdot [-NK [x \ln x + (1-x) \ln(1-x)]] \quad 4 \end{aligned}$$

$$6.13 \quad \frac{P(n)}{P(p)} = e^{-\Delta E/kT} = e^{-(\Delta m)c^2/kT} = 0.86$$

$$\text{Fraction of neutron} \quad \frac{86}{186} = 0.46$$

$$\text{" " proton} \quad \frac{100}{186} = 0.54$$

6.14 The relative probability of two states S_1 and S_2 is

$$\frac{P(S_2)}{P(S_1)} = \frac{e^{-E(S_2)/kT}}{e^{-E(S_1)/kT}} = e^{-mgz/kT}$$

z is the height altitude of state S_2 .

So the density ~~are~~ is proportional to state probability

$$P(z) = P(0) e^{-mgz/kT}$$

6.20

a)

$$\frac{1 - x \frac{1 + x + x^2 + \dots}{1 - x}}{x + 0x^2 + 0x^3 + \dots - x - x^2} = \frac{x^2 + 0x^3 + \dots - x^2 - x^3}{x^3 + \dots}$$

$|x| < 1$ to make

sure the series converging

b) $Z = \sum_s e^{-\beta E(s)}$ $E(s) = n\varepsilon$ $n=0, 1, 2, \dots$

$$= \sum_s e^{-\beta n\varepsilon} = \frac{1}{1 - e^{-\beta\varepsilon}}$$

c) $\bar{E} = -\frac{1}{Z} \frac{\partial Z}{\partial \beta} = \frac{\varepsilon}{e^{\beta\varepsilon} - 1}$

d) $U = N\bar{E} = \frac{N\varepsilon}{e^{\beta\varepsilon} - 1}$

e) $T \gg 1$ $\beta = \frac{1}{kT} \ll 1$ $U = \frac{N\varepsilon}{\beta\varepsilon} = \frac{N}{\beta} = \frac{N}{kT} \Rightarrow C = NK$

$T \rightarrow 0$ $\beta \gg 1$

$$C = \frac{\partial U}{\partial T} = \frac{\partial U}{\partial \beta} \cdot \frac{\partial \beta}{\partial T} = \frac{N\varepsilon^2}{kT^2} \frac{e^{\varepsilon/kT}}{(e^{\varepsilon/kT} - 1)^2}$$

Replace $\frac{G}{kT} = x$ when $T \rightarrow 0$ $x \rightarrow \infty$

$$\frac{C}{Nk} = \frac{x^2 e^x}{(e^x - 1)^2} = \frac{\left(\frac{x^2}{e^x}\right) \rightarrow 0 \text{ as } x \rightarrow \infty}{(1 - e^{-x})^2 \rightarrow 1 \text{ as } x \rightarrow \infty} \rightarrow 0$$

32 14/14

2 a) $\bar{X} = \sum_x x P(x) = \sum_x x \frac{e^{-\beta u(x)}}{Z} = \frac{\int x e^{-\beta u(x)} dx}{\int e^{-\beta u(x)} dx}$

b) At local minimum $\left. \frac{du}{dx} \right|_{x_0} = 0$

3

$$\bar{X} = \frac{\int_{-\infty}^{+\infty} x e^{-\beta [u(x_0) + a(x-x_0)^2]} dx}{\int_{-\infty}^{+\infty} e^{-\beta [u(x_0) + a(x-x_0)^2]} dx} = \frac{\int_{-\infty}^{+\infty} x e^{-\beta a(x-x_0)^2} dx}{\int_{-\infty}^{+\infty} e^{-\beta a(x-x_0)^2} dx}$$

$$= \frac{\int_{-\infty}^{+\infty} (y+x_0) e^{-\beta a y^2} dy}{\int_{-\infty}^{+\infty} e^{-\beta a y^2} dy}$$

odd term $\int_{-\infty}^{+\infty} y e^{-\beta a y^2} dy = 0$

$$= \frac{x_0 \int_{-\infty}^{+\infty} e^{-\beta a y^2} dy}{\int_{-\infty}^{+\infty} e^{-\beta a y^2} dy} = x_0 \quad \text{where } y = x - x_0.$$

4 c) $e^{-\beta u(x)} = e^{-\beta [u(x_0) + ay^2 + by^3]} \approx e^{-\beta u(x_0)} e^{-\beta ay^2} [1 - \beta by^3]$

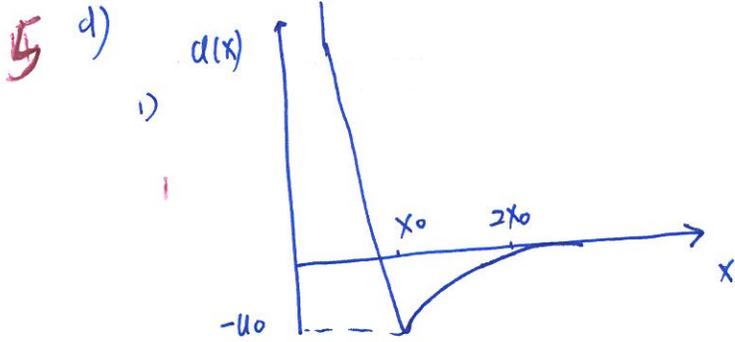
$$\bar{X} = \frac{e^{-\beta u(x_0)} \int (y+x_0) e^{-\beta ay^2} [1 - \beta by^3] dy}{e^{-\beta u(x_0)} \int e^{-\beta ay^2} [1 - \beta by^3] dy}$$

drop all odd terms

$$\bar{X} = \frac{\int [x_0 - \beta by^4] e^{-\beta ay^2} dy}{\int e^{-\beta ay^2} dy} = x_0 - \frac{3}{4} \frac{b}{a^2} kT$$

(see Appendix B)

$$\alpha = \frac{1}{\bar{X}} \frac{\partial \bar{X}}{\partial T} \approx -\frac{3b}{4a^2} \frac{k}{x_0}$$



2) $\frac{du}{dx} = 0 \Rightarrow u_0 [X_0^{12}(-12)X^{-13} - 2X_0^6(-6)X^{-7}] = 0 \Rightarrow \boxed{x = X_0}$

$u(X_0) = \boxed{-u_0}$

3) $\frac{d^2u}{dx^2} = u_0 [(-12)(-13)X_0^{12}X^{-14} - 2(-6)(-7)X_0^6X^{-8}] ;$

$\frac{d^3u}{dx^3} = u_0 [(-12)(-13)(-14)X_0^{12}X^{-15} - 2(-6)(-7)(-8)X_0^6X^{-9}]$

$a = \frac{1}{2} \frac{d^2u}{dx^2} \Big|_{X_0} = \boxed{\frac{36u_0}{X_0^2}}$

$b = \frac{1}{6} \frac{d^3u}{dx^3} \Big|_{X_0} = \boxed{-\frac{252u_0}{X_0^3}}$

$\alpha = -\frac{3}{4} \frac{b}{a^2} \frac{K}{X_0} = \frac{7}{48} \frac{K}{u_0} = \frac{1.26 \times 10^{-5} \text{ eV/K}}{u_0}$

For argon $u_0 = 0.10 \text{ eV} \Rightarrow \boxed{\alpha = 0.0013 \text{ K}^{-1}}$

higher than $\alpha = 0.0007 \text{ K}^{-1}$

6.4
10/10 The number of velocity vectors corresponding to speed v is proportional to $2\pi v$.

$$\Psi(v) = c \cdot 2\pi v e^{-mv^2/2kT}$$

To normalize it,

$$1 = 2\pi c \int_0^{\infty} v e^{-mv^2/2kT} dv = 2\pi c \left(\frac{2kT}{m}\right) \cdot \frac{1}{2}$$

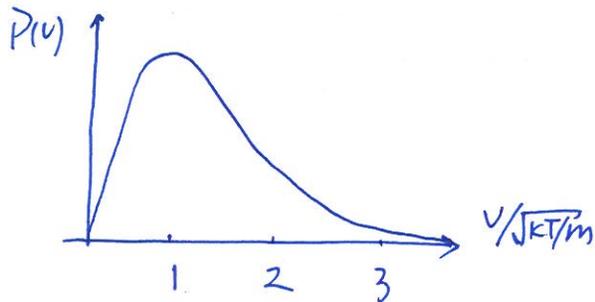
$$\Rightarrow c = \frac{m}{2\pi kT}$$

$$P(v) = \left(\frac{m}{2\pi kT}\right) 2\pi v e^{-mv^2/2kT}$$

$v \rightarrow 0$. $2\pi v$ dominate \rightarrow linear

$v \rightarrow \infty$ exp dominate

2 In 3D. it's parabolic
Same as 3D



2 most likely vector $\vec{v} = 0$

most likely speed

$$0 = \frac{dP(v)}{dv} \Rightarrow e^{-mv^2/2kT} - (v) \left(\frac{mv}{kT}\right) e^{-mv^2/2kT} = \left(1 - \frac{mv^2}{kT}\right) e^{-mv^2/2kT}$$

$$2 \quad v_{\text{prop}}^{\text{max}} = \sqrt{kT/m}$$